Project 4: Unsteady Turbulent Flow around a Cylinder

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This document is set in landscape to show the tables without special formattling. The entire source code can be found on `https://gitlab.mn.tu-dresden.de/s1140568/fluiddynamicsproject4`.

1. Mesh

The mesh has two zones: one around the cylinder and a wider one for the remainder of the domain. The zone around the cylinder is a square with arked edges. Those arks distribute the angles about equally around the corners of the square and distribute the grading partially into the outer zone.

In the simulations with a previous version with 20 cells in the radial direction, the heat around the cylinder could not be resolved well at Reynold number of 10^4 and 10^5. Therefore this radial resolution was increased to 40 (times N).

In an earlier version, the grading around the cylinder is chosen in a way that maximizes the grading while keeping the cell size ratio at 105 %. So the grading increasd with higher number of cells but this got too extreme leading to extremely high Courant numbers and bad cell size ratios at the square ark edges. So the grading was fixed to 45.

Since the cells close to the cylinder should be smaller but also dictate the size in one dimension over, under, left and right of the square part, outer cells in those four directions get very long and thin.

The square side length is 3 times the cylinder diameter while the domain height L is 20 the cylinder diameter. Since calculation time is already very high for the coursest of the meshes, a higher L does not sound feasible. The mesh at $N = 1$ is displayed in figure 1.

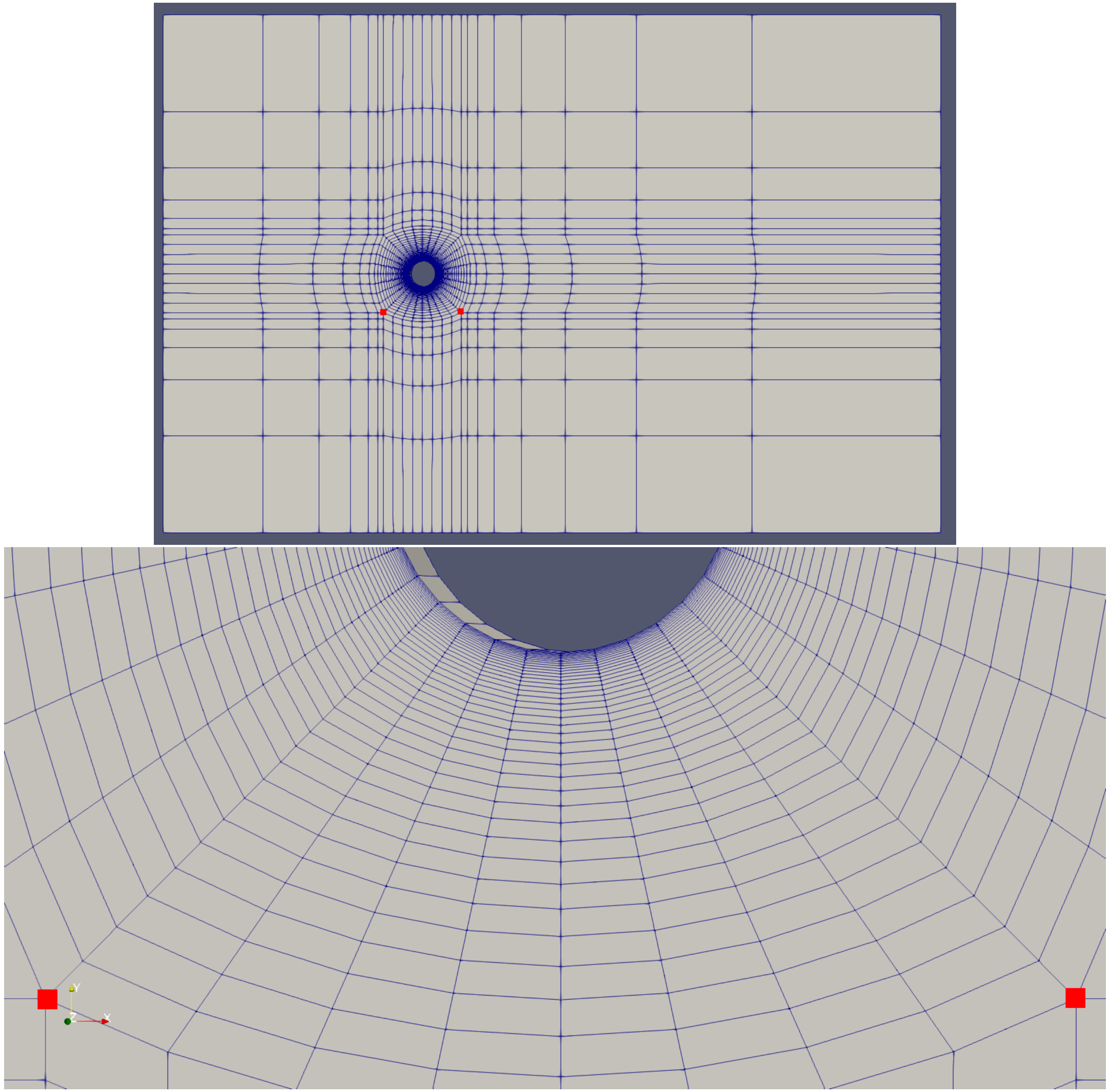

Figure 1: The mesh at $N = 1$. As mentioned in the description the resolution at the cylinder wall is very high and therefore a second close-up view of the cylinder is displayed here (which is still too big). For orientation the corners of the arked square are marked red.

2. Boundary values

The turbulence parameters all have the condition `inletOutlet` for the inlet and the outlet. This is supposed to be an uniform condition at inflow (at inlet) and a zero gradient condition on the outlet.

The pressure is calculated and the actual pressure boundary conditions are given for the adjusted pressure p_{rgh}. All other values have cyclic conditions on the top and bottom (`type cyclic;`) and empty conditions for front and back to ensure a 2D-problem.

The other boundary conditions are listed in table **??**.

Table 1: The boundary conditions for all value and boundaries except top, bottom, front and back.

Value	std. value	inlet	outlet	cylinder
α_t	0	inletOutlet	inletOutlet	`alphatJayatillekeWallFunction`
ν_t	0	inletOutlet	inletOutlet	`nutkWallFunction`
k	$4 \cdot 10^{-5}$	inletOutlet	inletOutlet	`kqRWallFunction`
ε	$2 \cdot 10^{-8}$	inletOutlet	inletOutlet	`epsilonWallFunction`
ω	$5 \cdot 10^{-4}$	inletOutlet	inletOutlet	`omegaWallFunction`
T	300	Dirichlet T_∞	$\nabla T = 0$	Dirichlet T_w
U		Dirichlet U_{in}	`fluxCorrectedVelocity`	`noSlip`
p_{rgh}		`fixedFluxPressure`	Dirichlet 0	$\nabla p_{rgh} = 0$

3. Parameters

The temperatures were set to $T_\infty = 300\,\mathrm{K}$ and $T_w = 400\,\mathrm{K}$. The laminar Prantl number was set to 0.7 while the turbulent Prantl number was 1.0.

4. Solvers

For p_{rgh} preconditioned conjugate gradient (PCG) solver with DIC-smoothed GAMG-preconditioner. The preset GAMG solver with `DICGaussSeidel` preconditioner was used before and the change did not affect the runtime significantly.

For the other values a smooth solver with symmetric Gauss-Seidel smoother was used. The preset PBiCGStab solver was used before and the runtime was not significantly affected by the change.

5. Schemes

For the time derivative a `backward` scheme was used. Gauss linear was used as a gradient Scheme while `Gauss limitedLinear 1` was used as all divergence Schemes (in the vector version for U.) The wall distance (`wallDist`) is calculated via a meshWave.

6. Reynolds number

The Reynolds number is defined as

$$\mathrm{Re}_d = \frac{ud}{\nu} \implies u = \frac{\mathrm{Re}_d\,\nu}{d} = \frac{\mathrm{Re}_d}{2000} = 5 \cdot 10^{-4}\,\mathrm{Re}_d\,.$$

In the default case $d = 2\,\mathrm{m}$, $\nu = 10^{-3}\,\frac{\mathrm{m}^2}{\mathrm{s}}$. The inflow velocity is chose in a way that the desired Reynold numbers are reached, see table 2.

7. Literature

Since the studied geometry is pretty simple, it has been studied numerous times before. For the Nusselt number Churchill and Berstein found the following formula in terms of Re_d and Pr with an accuracy of about 20 %:[1]

$$\overline{\mathrm{Nu}_d} = 0.3 + \frac{0.62\sqrt{\mathrm{Re}_d}\,\sqrt[3]{\mathrm{Pr}}}{\sqrt[4]{1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{\frac{2}{3}}}} \cdot \left(1 + \left(\frac{\mathrm{Re}_d}{282000}\right)^{\frac{5}{8}}\right)^{\frac{4}{5}}$$

The results of this equation are listed in table 2.

For reference values for the drag coefficiant we can refer to the paper Nakbas, Wanib, and Allm [NWA07] which solved the same problem for $\mathrm{Re}_d = 100, 1000$ and 3900. The values for 100 and 1000 are inserted into table 2 for comparison.

Low Reynolds number Nakbas, Wanib, and Allm [4.1.1 The laminar flow NWA07, p. 5] reports that at $\mathrm{Re}_d = 100$ a steady state is reached after 15 s. This was not confirmed by my experiments: the visual inspection indicated a stabilisation around 60 s to 80 s. The development of the drag coefficiant over time indicates stabilisation at around 130 s, for $N = 4$ even later. Therefore this experiment was run until $t = 150\,\mathrm{s}$. The $|\nabla T|$ did not stabilise within 150 s though as visible in figurefig:gardTNotStable.

Medium Reynolds number Nakbas, Wanib, and Allm [4.1.2 The turbulent flow NWA07, p. 9] noted that for $\mathrm{Re}_d = 1000$ the necessary time steps need to be very small and therefore the maximal time length could not be long. They observed periodical fluctuations of the drag coefficiant stabilising at around 140 s. My experiment did not do the same but decreases within the time range of 150 s and did not stabilise during this time. Therefore the simulation time was increased to 230 s where it was getting closer to a stable value for $N = 1$ but showed signs of either divergence or fluctuation for $N = 2$.

Nakbas, Wanib, and Allm [NWA07] introduced an additional pertubation that should decrease the time the simulation needs to reach an interesting state. This was not done in this experiment. The time step for $N = 2$ to guarantee a maximal Courant number of 1 was about 0.05 with about 5 time steps calculated per (real-world) second. There was not enough time to calculate the case for $N = 4$.

High Reynolds number Due to the long running time of simulations with $\mathrm{Re}_d \in \{10^4, 10^5\}$ only one experiment with $N = 1$, $\mathrm{Re}_d = 10^4$ and $t_{\max} = 20\,\mathrm{s}$ was calculated. The development of drag coeffciant and $|\nabla T|$ looked as if it stabilised but due to the short running time this is uncertain.

[1] `https://en.wikipedia.org/wiki/Churchill%E2%80%93Bernstein_equation`

8. Measurements

The drag coefficiant was calculated with the OpenFOAM intrinsic function `forceCoeffsIncompressible` with the reference area $Aref$ of $2 \cdot r \cdot l = 2\,\mathrm{m}^2$ where l is the thickness of the domain and r the radius of the cylinder.

The average temperature gradient of the cylinder was calculated with the OpenFOAM instrinsic post processing functions `grad(T)`, `mag(grad(T))` and `patchAverage(name=cylinder,mag(grad(T)))`. We need $|n \cdot \nabla T|$ instead of $|\nabla T|$ but in this case those two values are equal since ∇T can be split into the tangential and normal parts and the tangential part must be zero since the temperature of the cylinder is equal everywhere.

The results are summerized in table 2.

Table 2: Scalar results of the numerical experiments. ref-Nu_d is the Nusselt number according to the Churchill-Bernstein equation. Reference drag coefficiants are taken from Nakbas, Wanib, and Allm [Table 1 NWA07, p. 14]. Value with (*) have an high uncertainty because they have not stabilised yet.

Re_d	u_{in}	# cells	Δt	$t_{\max}$	C_D	$\vec{n} \cdot \nabla T\|_{\mathrm{cyl}}$	Nu_d	ref-C_D	ref-Nu_d
10^2	0.05	1656	0.2 s	150 s	$8.44 \cdot 10^{-2}$	$2.84 \cdot 10^2$	5.68	1.245	5.1
10^2	0.05	6624	0.2 s	150 s	$7.18 \cdot 10^{-2}$	$2.69 \cdot 10^2$	5.38	1.245	5.1
10^2	0.05	26496	0.2 s	150 s	$5.66 \cdot 10^{-2}$	$2.3 \cdot 10^2$	4.6	1.245	5.1
10^3	0.5	1656	0.12 s	230 s	4.84	$7.9 \cdot 10^2$	15.8	0.995	15
10^3	0.5	6624	0.2 s	230 s	6.39	$7.5 \cdot 10^2$(*)	15	0.995	15
10^4	5	1656	0.009 s	20 s	4.58	$2.58 \cdot 10^3$	51.6	—	15

9. Comparison and conclusion

For the relatively low Reynolds number of $\mathrm{Re}_d = 100$ this setup produced results that did not fit to the values in the literature: the drag coefficiant is off by one to two orders of magnitude, it did not stabilise within the same time frame. Only the Nusselt number was in the same order of magnitude as predicted by the Churchill-Bernstein equation but the value varied greatly between different mesh densities and did not stabilise during run time. So the setup is not suitable.

For $\mathrm{Re}_d = 1000$ the simulation was not able to exhibit the regular fluctuations observed by Nakbas, Wanib, and Allm [NWA07] and the drag coefficiant was off by a factor of five (closer than for $\mathrm{Re}_d = 100$ but the Nusselt number was again close to the Churchill-Bernstein equation value.

For higher Reynold numbers $\mathrm{Re}_d \in \{10^4, 10^5\}$) the computation became unfeasible slow because the Courant numbers dictate time steps at the order of magnitude of 0.008 ($N = 1$, $\mathrm{Re}_d = 10^4$) to 0.0008 ($\mathrm{Re}_d = 10^5$) and the stabilisation times dictate running times of at least 30 s (according to turbulent case in Nakbas, Wanib, and Allm [NWA07]). Therefore those cases could not even be compared to the literature.

For all Reynold numbers the different meshes showed quite different results. That gives no confidence for the reliability of the coarse meshes. This setup for numerical computation for unsteady flow around a cylinder in 2 dimensions is suitable to calculate the rough estimates on the Nusselt number but does not give reliable results for drag coefficiants or fluctuation frequencies and does not scale to denser meshes or higher turbulence, indicated by higher Reynold numbers. The author cannot tell if that is due to an inferior mesh, wrong boundary conditions, unsuitable time stepping or solver and scheme choice. On each of those variables except boundary conditions some variations were tried without noticeable improvement.

Appendices

A. Additional figures

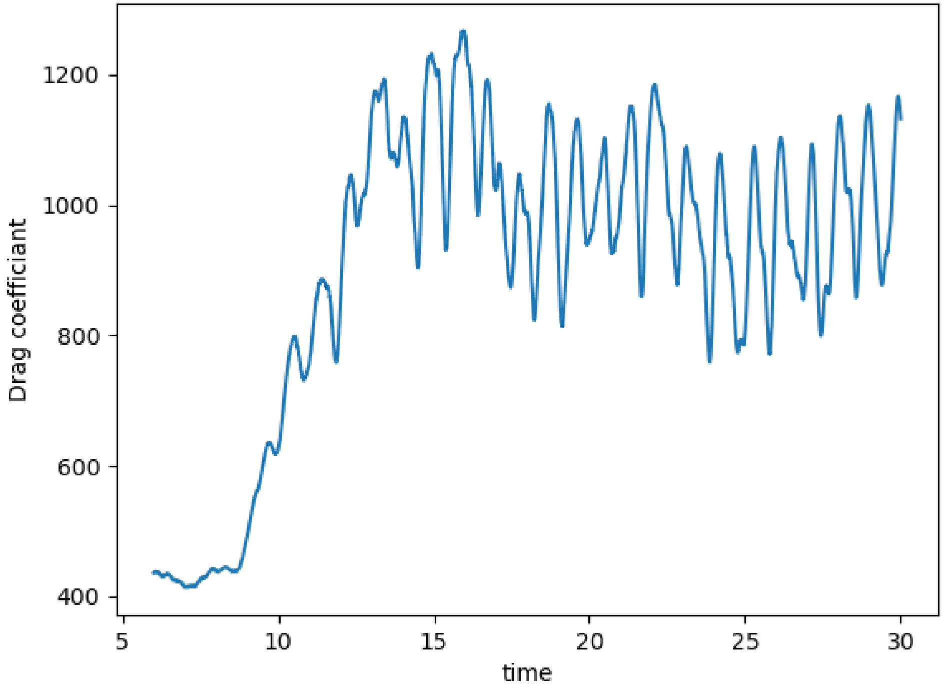

Figure 2: The drag coefficiant at a Reynolds number of 10^3 ($u = 5\,\frac{\mathrm{m}}{\mathrm{s}}$) and $N = 2$ (6624 cells). The Courant number rose to about 3.4 and it can be seen that this resulted in highly inaccurate and useless results. For higher Re_d this became worse (also for $N = 1$).

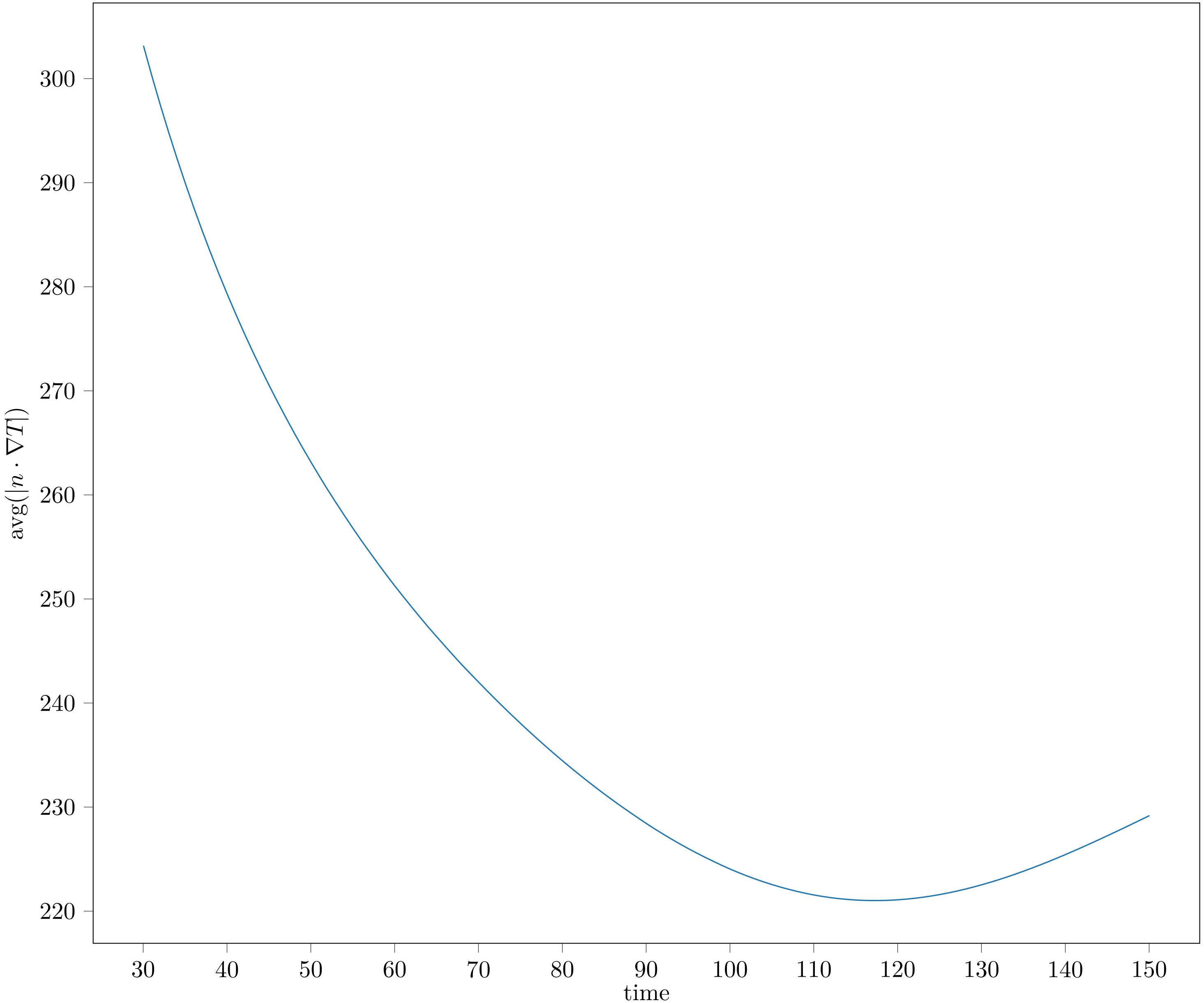

Figure 3: Development of $|\nabla T|$ over time at $\mathrm{Re}_d = 100$ and $N = 4$ (26496 cells). We see no stabilisation. Therefore this value is not reliable. For $N = 1$ and $N = 2$ the curves look different but not stabilising either.

References

[NWA07] Bakzuzan Nakbas, Baokut Wanib, and Abdul Allm. "Numerical investigation of unsteady flow past a circular cylinder using 2-D finite volume method". In: *Journal of Naval Architecture and Marine Engineering* (June 2007). URL: https://www.banglajol.info/index.php/JNAME/article/download/914/980/ (cit. on pp. 4, 5).